Review For Exam 1

The directions for the exam are as follows:

"WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!"

1. In other words, the exam consists of 10 core problems and 2 extra-credit problems. If you wish, you can do all the 12 problems, but your score will only add up to 100 points. Partial credit will be given.

2. Also remember that you are allowed to use a scientific calculator.

3. When you are studying for this exam, be sure to work through sections that you know least of all first.

4. Odd exercises have solutions at the back of your textbook.

Warning! Be sure to work on ALL exercises below that are marked in red. Do ALL the problems on the review list to insure a perfect mastery of the topic.

Section 2.3

- Be able to compute limits by relying on the limit laws (P. 106, Exercises **3, 5, 7**)
- Be able to compute limits of algebraic functions and to justify your calculations if necessary. (P. 107, Exercises **11-31** [odd])
- Understand the idea behind the **Squeeze Theorem** (P. 107, Exercises **37-40**)

Section 2.4 (Extra-Credit)

- Know how to prove that a limit exists using the $\delta - \epsilon$ argument. (P.117, Exercises **19-27**)
- Prove all the limit laws using the $\delta - \epsilon$ argument.
- At this level, the limit laws are the only useful consequence of the $\delta - \epsilon$ definition of the limit. Understanding the precise definition of limit is indispensable when dealing with more subtle mathematical concepts.
- Be able to analyze infinite limits graphically (P. 67, Exercises **9, 11, 15**)
- Be able to determine infinite limits analytically (P. 68-69, Exercises **17-27** [odd])
- Be able to identify vertical asymptotes (P. 69, Exercises, **29, 31, 33**)

Section 2.5

- Is $\lim_{x \to a} f(x)$ the same thing as f (a) for *any* function f? What exactly is continuity?
- Know how to determine whether the function is continuous at the given point (P. 128-129, Exercises **17-21** [odd], **39, 41, 42, 45, 46**)

- Why are polynomials continuous? Are rational functions continuous? Explain.
- Be able to apply the Intermediate-Value Theorem (P. 129, Exercises **51-55** [odd])
- Suppose a is a positive real number. Let $f(x) = \begin{cases} x^3 & if \quad 0 \le x < a \\ \sqrt[3]{x} & if \quad x \ge a \end{cases}$. What must be the value of a if f(x) is a continuous function?
- **Possible Extra-Credit** Show that every continuous function $f: [0,1] \to [0,1]$ must square at least one number. That is, show that for some number c, $f(c) = c^2$ [Hint: Use the Intermediate-Value Theorem]

Section 2.6

- Be able to compute basic limits at infinity (P. 141, Exercises, **15-31** [odd])
- Be able to find horizontal and vertical asymptotes (P. 141, Exercises **41-45** [odd])

Section 2.7

- What are derivatives? Why are they important? What motivates the formulas that define them?
- Compute $f'(a)$ (P. 151 Exercises **27-31** [odd])
- Identify the function corresponding to the given limit (P. 151 Exercises **33-37** [odd])
- Find an equation of the tangent line to the curve. (P. 150 Exercises **5-7** [odd])

Section 2.8

- Know how to find the derivative of the function by applying either the formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ or $\lim_{z\to x} \frac{f(z)-f(x)}{z-x}$.
- How can a function fail to be differentiable? (Read P. 159)
- What is the relationship between differentiability and continuity? (Read P. 158)
- Given the graph of f, how can we draw the graph of f'? (Read P. 154-155). (P.162, Exercises **3, 9, 11**)

Section 3.1

- Be able to use basic derivative shortcuts to quickly compute derivatives. (P. 181, Exercises **3-25** [odd])
- **Possible Extra-Credit** Prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for every positive integer n.
- **Possible Extra-Credit** Prove that $\frac{d}{dx}\left(\sqrt[n]{x}\right) = \frac{d}{dx}\left(x^{1/n}\right) = \frac{1}{n}x^{\frac{1}{n}-1}$ for every positive integer n.

Section 3.2

- Be able to use product, quotient, and more basic rules to quickly compute derivatives. (P. 189, Exercises **3-25** [odd])
- **Possible Extra-Credit** Show that if a function $f(x)$ is differentiable at $x = a$, then it must also be continuous at $x = a$.
- **Possible Extra-Credit** Prove the derivative product rule.
- **Possible Extra-Credit** Prove the derivative quotient rule.

Section 3.3

- From knowledge that $\lim_{h\to 0}\frac{Sin(h)}{h} = 1$ and $\lim_{h\to 0}\frac{Cos(h)-1}{h} = 0$, be able to compute kindred limits. (P. 198, Exercises **39-47** [odd])
- From knowledge that $\frac{d}{dx}\big(Sin(x)\big) = Cos(x)$ and $\frac{d}{dx}\big(Cos(x)\big) = -Sin(x)$, be able to compute derivatives of other trig functions (P. 197, Exercises **1-15** [odd])
- Find equations of tangent lines for trigonometric functions. (P. 197, Exercises **21-24**)
- **Possible Extra-Credit** Establish the derivative of $\sin x$ from the definition.
- **Possible Extra-Credit** Establish the derivative of $\cos x$ from the definition.
- **Possible Extra-Credit** Establish the formulas for the derivatives of the other 4 trigonometric functions from your knowledge that $\frac{\mathrm{d}}{\mathrm{dx}}\sin x = \cos x$ and $\frac{\mathrm{d}}{\mathrm{dx}}\cos x = -\sin x$.

Practice Exams

- Review the problems on the "Concentrated Review of Trouble Topics for Exam 1".
- Solve all the problems on practice exams 1.1-1.3 under the time constraint of 2 hours.